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To investigate the relationship between confidence limits of individual means and confidence limits of the difference between two means using Mathcad.

Investigation prompted by Thiessen, H. (1996) *Measuring the Real World*, Wiley, Chichester and New York. Section 11.7.1.

Assumptions

Assume that the true unknown variances are the same in the two groups.
Two sample t test will be used.
Test will be applied two-sided
Assume sample size, *n*, is the same in the two groups.
Let *r* be the ratio of the sample variance in group 2 (sv_2) to group 1 (sv_1).
Assume, without loss of generality that the variance in group 1 is 1.

υ_{single} are degrees of freedom for a single sample
υ_{pooled} are degrees of freedom for the test for the difference between means

α= significance level for test of difference.

Set parameters and define relationships

$\alpha := 0.05$

$r := 0.2, 0.3 .. 5$ $\quad sv_1 := 1$ $\quad sv_2(r) := r \cdot sv_1$ $\quad n := 100$

$\nu_{single}(n) := n - 1$ $\quad \nu_{pooled}(n) := 2 \cdot (n-1)$ $\quad var_{pooled}(r) := \dfrac{sv_1 + sv_2(r)}{2}$

Begin calculations

Calculate critical value of t distribution for use with standard error of difference

$$crit_{diff}(n) := qt\left(1 - \frac{\alpha}{2}, \nu_{pooled}(n)\right)$$

Examples

$crit_{diff}(31) = 2$ $\quad$ $crit_{diff}(1000) = 1.961$

Calculate differences between sample means if results just critical.

$$mean(n,r) := crit_{diff}(n) \cdot \sqrt{var_{pooled}(r) \cdot \left(\frac{2}{n}\right)}$$

Example

$mean(1000,1) = 0.088$

Solve for t values for individual confidence intervals

$$t(n,r) := \frac{mean(n,r)}{\sqrt{\frac{sv_1}{n}} + \sqrt{\frac{sv_2(r)}{n}}}$$

Example

$t(n,1) = 1.394$

Solve for confidence level associated with individual means

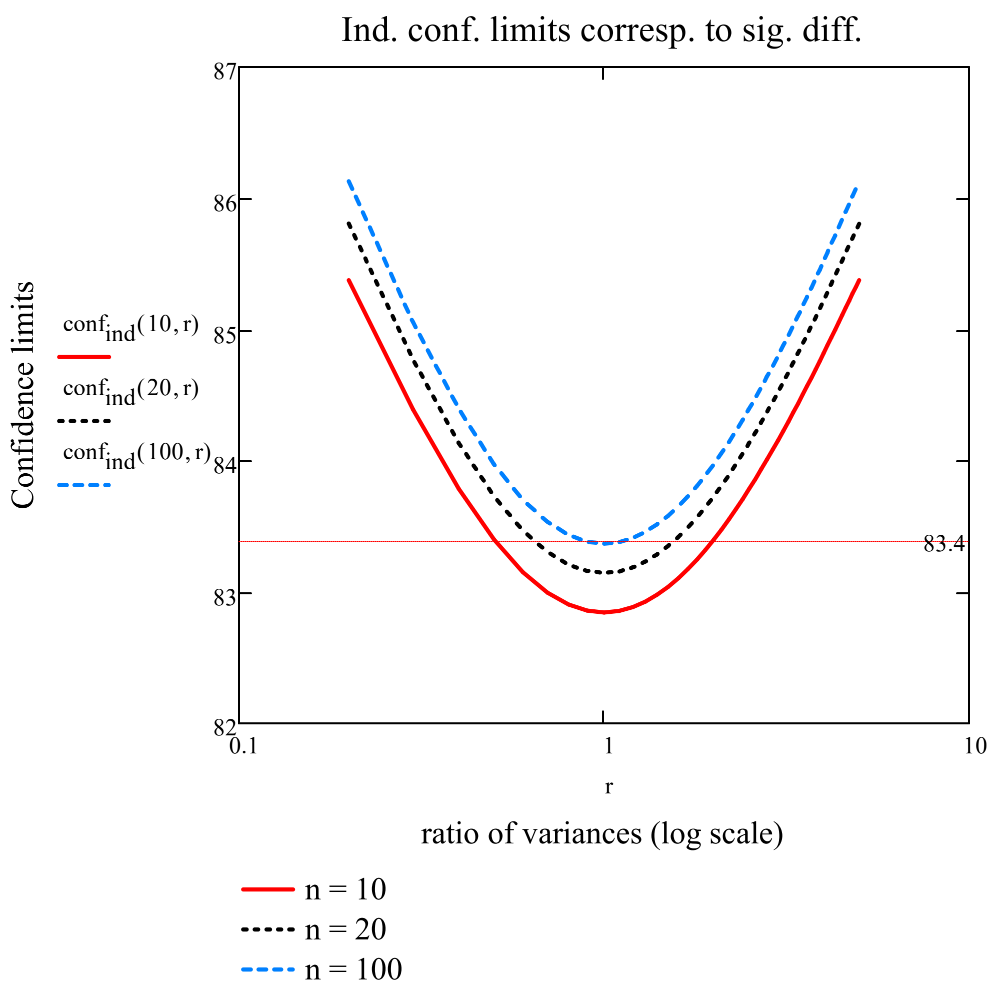
$$conf_{ind}(n,r) := 100 \cdot \left[1 - 2 \cdot \left(1 - pt\left(t(n,r), \nu_{single}(n)\right)\right)\right]$$

$pt\left(t(n,1), \nu_{single}(n)\right) = 0.917$

$conf_{ind}(n,1) = 83.369$ $\quad$ $conf_{ind}(1000,1) = 83.417$

Plot results

$\alpha = 0.05$ n is sample size, assumed same in both groups



Explanation of graph

What the curves show is the individual levels of confidence (*Y* axis) such that if each of the two means had confidence limits with these levels and these limits did not overlap, the difference would be significant but if they did overlap the difference would not be. The *X* axis shows the ratio between the first sample variance and the second and the critical individual levels of confidence are plotted as a function of these. Different curves are plotted for different sample sizes

Discussion

Note that there is a slight dependence on sample size and a rather stronger one on the observed ratio *r*. However, it seems that in most cases, if the 84% confidence intervals do not overlap the difference is significant at the 5% level. (95% limits for the difference will exclude 0.)